ON NEW THREE-TERM RECURRENCE RELATIONS FOR THE 3-j COEFFICIENT

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Abstract: Six new recurrence relations have been derived for the Clebsch-Gordan coefficient, also referred to as the Wigner 3-j coefficient. These are a consequence of the recurrence relations for the $_3F_2(\mathbf{a};\ \mathbf{b};\ z)$ derived recently by Tamara Antonova, Roman Dmytryshyn and Serhii Sharyn(2021).

Keywords and Phrases: Generalized hypergeometric series, Angular momentum coupling coefficient, Clebsch-Gordan, or 3-j coefficient, recurrence relations.

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1. Introduction

The well-known 3-j coefficient (or the Clebsch-Gordan coefficient), in Quantum Theory of Angular Momentum (QTAM) [8] is defined as:

$$
\begin{aligned}
\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} &= \delta_{m_1+m_2+m_3,0}\ (-1)^{j_1-j_2-m_3}\ \Delta(j_1 j_2 j_3) \\
&\quad \times \prod_{i=1}^{3} [(j_i+m_i)!\ (j_i-m_i)!]^{1/2} \\
&\quad \times \sum_t (-1)^t\ [\ t! \prod_{k=1}^{2} (t-\alpha_k)!\ \prod_{\ell=1}^{3} (\beta_\ell - t)!\ ]^{-1}.
\end{aligned} \tag{1}
$$